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THE EFFECT OF INSULATING VERTICAL WALLS ON THE ONSET OF MOTION IN A FLUID HEATED FROM BELOW

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Abstract—The initiation of natural convection in a fluid confined above and below by rigid perfectly conducting surfaces and laterally by rigid perfectly insulating vertical walls that form a rectangular shape is examined. The linearized perturbation equations are obtained and by appropriate non-dimensionalization, they are reduced to an eigenvalue problem. The Rayleigh number is the eigenvalue and is a function of two aspect ratios (width/height and depth/height). The problem associated with satisfying the no-slip boundary conditions on all surfaces is surmounted by using the Galerkin method. Results are obtained for aspect ratios ranging from $\frac{1}{8}$ to 12. The results are compared with experiment and found to be in good agreement.

1. INTRODUCTION

THE EFFECT of confining boundaries on convection in a fluid heated from below has been of interest to engineers for many years. It has only been in the past decade that serious attempts have been made to obtain criterion for the onset of convection. It is now well known that when the fluid is constrained by lateral walls, the required critical temperature gradient will be increased greatly. Ostrach and Pnueli [1] have used Pellew and Southwell's technique of separation of variables to predict the onset of convection for perfectly conducting walls in an approximate manner. Yih [2] has treated the infinite cylinder with perfectly insulating lateral wall, and Ostroumov [3] has given approximate relations for the case of an infinitely long cylinder with arbitrarily conducting side wall. Experimental measurements of the effect of lateral walls have been reported by Catton and Edwards [4] for the case where the lateral walls are nearly perfectly conducting or nearly perfectly insulating. Their data indicate that the approximations made heretofore (e.g. [1–3]) do not allow accurate predictions for the case of moderate aspect ratios. Kurzweg [5], Samuels and Churchill [6] and others have investigated the stability of an infinite roll. It has been shown by Edwards [7], among others, that the motion yielding the lowest critical temperature gradient is three dimensional even for the infinite slot.

Investigations of the fully confined fluid problem (rigid boundary conditions on all surfaces) that are of interest to the present work are those of Sherman and Ostrach [8], Davis [9], Catton [10] and Charlson and Sani [11]. Sherman and Ostrach obtained lower bounds on the Rayleigh number for arbitrary three dimensional enclosures and present results for several regular geometries.

Davis chose perfectly conducting walls and used the Galerkin method. In using the Galerkin method, however, he violated the Weierstrauss theorem, and his set of trial functions were not complete within the region of interest. This work has since been redone by Catton using a better set of trial functions. The results point out the benefits of using orthogonal functions. It is of interest to note that Sherman and Ostrach obtain a lower bound on the Rayleigh number of 3500 as compared to Davis or Catton who obtain an upper bound of 7000 for the perfectly conducting cube. Charlson and Sani have recently looked at low aspect ratio cylinders heated from below. They obtained upper and lower bounds for both perfectly conducting and perfectly insulating bounding walls. They used orthogonal functions to represent the velocity and temperature fields with the Galerkin method. The method used by Samuels and Churchill [6] was numerical and indicated the difficulties associated with just a two-dimensional problem. This work was followed by Azziz and Hellums [12] attempt at solving a three-dimensional problem.

The situation of interest to engineers is the case where the walls are insulators. Under these circumstances the change in heat transfer (increase in effective fluid conductivity) is much more marked and hence of much more interest. Wooding [13], Yih [2], Edwards [7], Edwards and Sun [14] have looked at this problem, but in all cases either allowed "free" (zero shear) top and bottom or vertical walls. This work satisfies the correct boundary conditions on all bounding surfaces and is not limited to narrow or tall cells as are the other researches in this area.

II. GOVERNING EQUATIONS

In the initial state, a quasi-incompressible (Boussinesq) fluid fills a rectangular region as shown in Fig. 1. The base of the rectangle is fixed at a temperature higher than the top and a linear temperature gradient is established in the fluid in the direction of the body force (the negative z axis). The initial velocity, temperature, and pressure distributions are given by:

$$\boldsymbol{v}_0 = 0, \nabla T_0 = \beta \boldsymbol{k}, \nabla P_0 = g(1 - \alpha\beta z)\boldsymbol{k} \quad (1)$$

where ρ is the mean fluid density, $\boldsymbol{k}$ a unit vector along the z axis, β the mean temperature gradient, α the volumetric thermal expansion coefficient and g the acceleration of gravity. In this particular problem, that instability sets in via a marginal stationary state was shown by Sherman and Ostrach [15]; hence terms with time derivatives will not appear in the equations

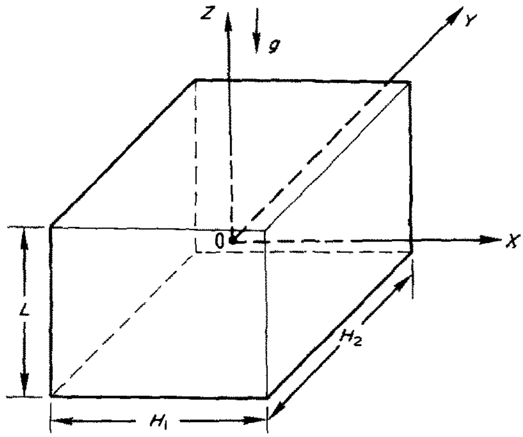

FIG. 1. The geometry and coordinate system of the rectangular region.

governing the perturbations. In dimensionless forms, the perturbation equations are (see [16])

$$\operatorname{div} \boldsymbol{v} = 0 \quad (2)$$

$$\nabla^2 \boldsymbol{v} + R\theta \boldsymbol{k} - \operatorname{grad} p = 0 \quad (3)$$

$$\nabla^2 \theta + w = 0 \quad (4)$$

where $\boldsymbol{v}$, θ and p are the velocity, temperature and pressure disturbances measured in units of κ/L, βL, and $\rho\nu\kappa/L^2$ respectively. The characteristic length L is the height of the rectangular region and ν and κ the kinematic viscosity and thermal diffusivity. The horizontal coordinates, x and y, are measured in units of the rectangle's height L. The Rayleigh number is defined

$$R = \frac{\alpha g \beta L^4}{\nu\kappa}. \quad (5)$$

The boundary conditions for this problem are

$$\boldsymbol{v} = 0 \text{ on } |z| = \tfrac{1}{2}, |x| = \tfrac{1}{2}H_1, |y| = \tfrac{1}{2}H_2 \quad (6)$$

$$\theta = 0 \text{ on } |z| = \tfrac{1}{2}, \frac{\partial\theta}{\partial x} = 0 \text{ on } |x| = \tfrac{1}{2}$$

$$H_1, \frac{\partial\theta}{\partial y} = 0 \text{ on } |y| = \tfrac{1}{2}H_2. \quad (7)$$

Equations (2) and (4) and the boundary conditions given by equations (6) and (7) constitute an eigenvalue problem for the Rayleigh number. The smallest eigenvalue is the desired critical Rayleigh number.

III. METHOD OF SOLUTION

The Galerkin method is readily adapted to a problem of this type. Kurzweg [5], Davis [8] and Charlson and Sani [11] have used this method. Kurzweg operated on equation (5) to eliminate pressure and in doing so the final secular determinant is not symmetric. He then solved the secular determinant iteratively and with its order being twice the number of trial functions used. Solving the problem this way presents difficulties in determining the contributions of each trial function to the composite. Davis used equations (3) and (4) with trial functions for the velocity that satisfy equation (2). This results in a secular determinant that is of a particular type. It is symmetric and its solution yields the critical Rayleigh number and the coefficients associated with the trial functions for velocity and temperature easily.

The interior orthogonality relations are written for the equations of motion (see [15])

$$\sum_{j}^{N} \int_{v} (\nabla^2 \boldsymbol{v} + R\theta \boldsymbol{k} - \operatorname{grad} p)_j \cdot \boldsymbol{v}_k \, \mathrm{d}V = 0 \qquad (8)$$

$$\sum_{j}^{N} \int_{v} (\nabla^2 \theta + w)_j \, \theta_k \, \mathrm{d}V = 0 \qquad (9)$$

where $\boldsymbol{v}_j$, θ_j and p_j are represented by

$$\boldsymbol{v}_j = a_j \boldsymbol{F}_j(x, y, z), \theta_j = b_j G_j(x, y, z),$$
$$P_j = C_j H_j(x, y, z). \qquad (10)$$

When the expressions given by equation (10) are substituted into equation (8) and (9) and the indicated integration is carried out, the pressure will vanish due to the solenoidal characteristic of $\boldsymbol{v}_j$ and there results

$$\sum_{j=1}^{N} \int_{v} |a_j \boldsymbol{F}_k \cdot \nabla^2 \boldsymbol{F}_j + R b_j G_j \boldsymbol{k} \cdot \boldsymbol{F}_k| \, \mathrm{d}V = 0 \qquad (11)$$

$$\sum_{j=1}^{N} \int_{v} |a_j G_k \boldsymbol{k} \cdot \boldsymbol{F}_j + b_j G_k \nabla^2 G_j| \, \mathrm{d}V = 0. \qquad (12)$$

The requirement that equations (11) and (12) have non-trivial solutions (a_j and b_j non-zero) is that the secular determinant be zero,

$$\det \begin{vmatrix} M_{11} & R M_{12} \\ M_{21} & M_{22} \end{vmatrix} = 0 \qquad (13)$$

where M_{11}, M_{12}, M_{21} and M_{22} are $N \times N$ matrices defined as

$$M_{11} = \int_{v} \boldsymbol{F}_k \cdot \nabla^2 \boldsymbol{F}_j \, \mathrm{d}V, M_{12} = \int_{v} \boldsymbol{k} \cdot \boldsymbol{F}_k G_j \, \mathrm{d}V$$
$$M_{21} = \int_{v} \boldsymbol{k} \cdot \boldsymbol{F}_j G_k \, \mathrm{d}V, M_{22} = \int_{v} G_k \nabla^2 G_j \, \mathrm{d}V. \qquad (14)$$

By choosing suitable expansion for $\boldsymbol{v}$ and θ, it is possible to obtain good approximations for the lowest values of R by truncating the secular determinant (13) at some finite number of terms. The j and k in equation (14) each indicate some x, y, z dependence of a trial function. The trial functions used are the beam functions and sine and cosine functions. The method used in selecting trial functions for the velocity is that of Davis. Davis takes advantage of the fact that superposing two dimensional motions (each satisfying continuity) can generate any three-dimensional motion if enough terms are incorporated into the expansion. For example one might choose trial functions so that

$$\frac{\partial u}{\partial x} + \frac{\partial w_1}{\partial z} = 0, \frac{\partial y}{\partial y} + \frac{\partial w_2}{\partial z} = 0.$$

This yields

$$w = w_1 + w_2, u = u, v = v$$

which is fully three dimensional motion and is the sum of two finite rolls. The various trial functions used to construct a solution are given in Appendix A.

The determinant given by equation (13) is of a special form that yields to a canonical correlation analysis between the two sets of variables $\boldsymbol{F}_j$ and G_j (or velocity and temperature). The requirement is that M_{22} and M_{11} are composed of different variables and that

$$M_{12} = \text{the transpose of } M_{21}.$$

Inspection of equation (14) indicate that this is the case. This is of particular importance if one wishes to obtain the eigenfunctions. It should be noted that the method used by Kurzweg (higher order equations) does not yield this special case whereas that of Davis does.

Equation (13) can be reduced to

$$\left| M_{22}^{-1} M_{21} M_{11}^{-1} M_{12} - \frac{1}{R} \mathrm{I} \right| = 0 \quad (15)$$

which is a $N \times N$ eigenvalue problem rather than a $2N \times 2N$ determinant whose zeros must be found.

The procedure used to solve the problem posed by equation (15) was to use the trial functions given in Appendix A to generate the matrices [equation (14)]. The number of trial functions used (N) was increased until six significant figures of accuracy were obtained in R. This was done for each of the sets of functions given in Appendix A and for various combinations of sets of functions. The characteristic Rayleigh number being the minimum one found. This procedure was carried out for horizontal dimensions H/L from $\frac{1}{8}$ to 12 where L is the height for perfectly insulating walls. The minimum Rayleigh number for the onset of motion is given in tabular form in Table 1 and in graphical form in Fig. 2.

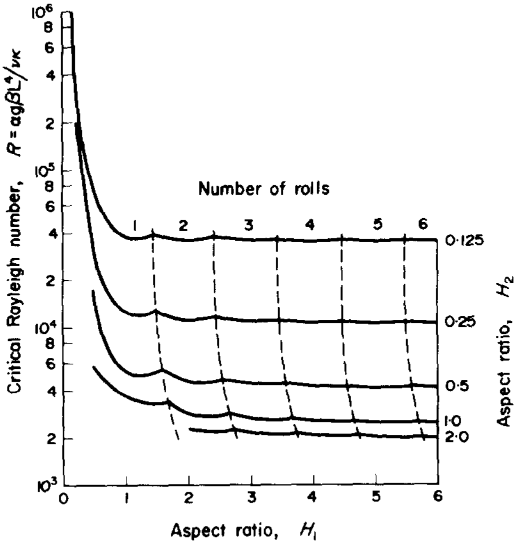

FIG. 2. The critical Rayleigh number for various aspect ratios.

IV. DISCUSSION OF RESULTS

The effect of adiabatic confining walls on the onset of natural convection has been determined for rectangular platforms for various aspect ratios. The Galerkin method (for this problem,

Table 1

H_1/H_2	0·125	0·25	0·50	1·00	2·00	3·00	4·00	5·00	6·00	12·00
0·125	3011718(1)									
0·25	333013(1)	203163(1)								
0·50	70040(1)	28452(1)	17307(1)							
1·00	37689(1)	11962(1)	5262(1)	3446(1)						
1·50	39798(2)	12540(2)	5341(1)	3270(1)						
2·00	36262(2)	11020(2)	4524(2)	2789(2)	2276(2)					
2·50	37058(3)	11251(3)	4567(2)	2754(2)	2222(2)					
3·00	35875(3)	10757(3)	4330(3)	2622(3)	2121(3)	2004(3)				
3·50	36209(4)	10858(4)	4355(3)	2609(3)	2098(3)	1978(3)				
4·00	35664(4)	10635(4)	4245(4)	2552(4)	2057(4)	1941(4)	1894(4)			
4·50	35794(5)	10666(5)	4261(4)	2545(4)	2044(4)	1927(4)	1878(4)			
5·00	35486(5)	10544(5)	4186(5)	2502(5)	2009(5)	1897(5)	1852(5)			
5·50	35556(6)	10571(6)	4196(5)	2498(5)	2001(5)	1888(5)	1842(5)			
6·00	35380(6)	10499(6)	4158(6)	2480(6)	1989(6)	1879(6)	1833(6)	1810(6)	1797(6)	
6·50	35451(7)	10518(7)	4165(6)	2447(6)	1984(6)	1871(6)	1826(6)	1803(6)	1789(6)	
12·00	35193(12)	10426(12)	4118(12)	2453(12)	1967(12)	1855(12)	1808(12)	1783(12)	1768(12)	1741(12)

identical to a Rayleigh–Ritz procedure) was used with trial functions constructed from a linear combination of a complete set or orthogonal coordinate functions. The trial functions were selected to allow for the possibility of fully three dimensional flow configurations. The minimum Rayleigh numbers were obtained for trial functions representing rolls whose axis are perpendicular to the longer dimension. The preferred orientation of rolls has been observed by Whitehead and Busse [18], and predicted by Davis [8] for perfectly conducting lateral boundaries.

The results of this analysis are presented in Table 1 for aspect ratios (H_1 and H_2) from 0·125 to 12. Selective results are presented graphically in Fig. 2. The stability curves, Fig. 2, obtained have kinks at various values of H_1/H_2. These kinks are caused by the number of rolls in the box increasing in jumps as the aspect ratio is increased. The kinks are much more pronounced than in this author's previous work [9] for the case of perfectly conducting walls. Charlson and Sani [11] also noted that the kinks were much more pronounced for non-conducting walls than for conducting walls. The kinks were also predicted by Davis [8] and Kurzweg [5]. It would be interesting to pursue this phenomena experimentally. What actually occurs in the box can be seen by inspecting Fig. 3.

Holding the smaller aspect ratio fixed and increasing the other causes the rolls in the box to stretch. The stretched roll requires a higher temperature difference to drive it (or a higher Rayleigh number). When the situation is such that increasing the number of rolls presents a situation that is more easily driven, then the number of rolls will increase. As can be seen from Fig. 3, an almost stagnant region appears in the end of the box. The stagnant region gradually gets strengthened and becomes the new roll as the larger aspect ratio is further increased.

The two dimensional channel has been investigated by Yih [2], Samuels and Churchill [6], Kurzweg [5] and Edwards and Sun [14] and are compared with this work in Table 2. The first three investigators assumed that the disturbance would be a two dimensional roll oriented with its axis parallel to the channel walls. Edwards and Sun assumed a fully three dimensional motion and found that the rolls were oriented with axis perpendicular to the channel wall as was found in this work. Their

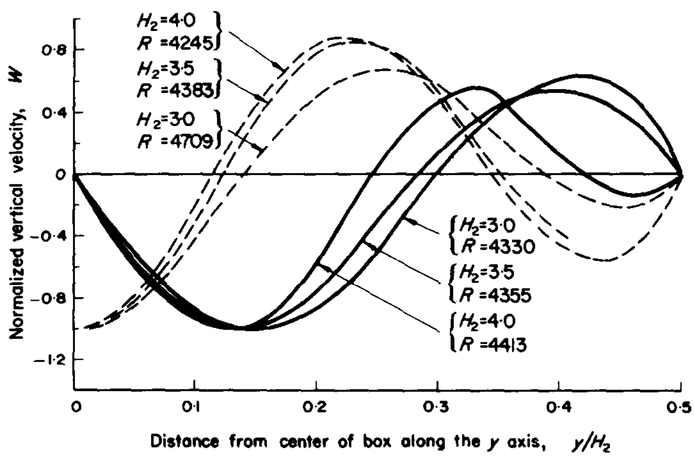

FIG. 3. Eigenfunctions in a region where the number of rolls changes from 3 to 4 when H_1 = 0·5.

Table 2

Width / Height	Yih	Samuels and Churchill	Kurzweg	Edwards and Sun*	This work (depth/height = 12·0)
0·25				14 550	10 426
0·50	11 280	11 600	12 120	5250	4118
1·00	1600	2460	2580	2705	2435
2·00	750	1840	2016	9940	1967
3·00			2030		1855

* Walls were slightly conducting.

solution assumed slip on one of the faces of the box and is limited to boxes far from square. The results are not dramatically different for aspect ratio greater than unity (except for Yih, who considered only the very small aspect ratio). For the smaller aspect ratios, there is a factor of two difference between the two approaches to the problem. It is obvious that the motion is three dimensional. Edwards and Sun obtained results a bit higher than the present results because their lateral walls were slightly conducting.

Catton and Edwards [4] have suggested that the Rayleigh number be evaluated from

$$R = \frac{64}{27}\left(\frac{9a^2}{16} + b^2\right)^3 / a^3 \tag{16}$$

for small aspect ratios. The vertical wave number, b, is given by

$$b = \pi + 0{\cdot}85 \tag{17}$$

and the horizontal wave number, a, is approximately the eigenvalue of

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) w + a^2 w = 0 \tag{18}$$

where x and y are coordinates in the horizontal plane and $w = 0$ on the lateral boundaries of the cell. For rectangles

$$a = \pi\left(\frac{4}{H_2^{\,2}} + \frac{1}{H_1^{\,2}}\right) \tag{19}$$

where H_2 is greater than H_1. On comparison of the values given by equation (16) with the present work, it was found that if H_1 is less than 1·0 and the ratio H_2/H_1 is less than 1·5, then less than 10 per cent discrepancy exists. The discrepancy decreases quickly for small values of H_1 and H_2. For larger values of H_1 and values of the ratio H_2/H_1, multiple rolls occur and the method deviates dramatically from the more exact values obtained herein.

A better approximation for the Rayleigh number can be obtained by using a one term approximation as a solution to equation (15). Doing so yields

$$R = 59\left(\frac{1}{H_2^{\,2}} + 1\right)\left(\frac{2{\cdot}98}{H_1^{\,2}} + \frac{11{\cdot}2}{H_2^{\,2}} - 0{\cdot}85\right) \tag{20}$$

when H_1 is less than H_2. This approximation is more accurate than that given by equation (16) but is still only valid when H_1 is less than 1·0 and the ratio H_2/H_1 is less than 1·5.

The only experimental data available were that of Edwards and Sun [14]. A comparison is made in Table 3. The present results are

Table 3

H_1	H_2	Rayleigh number: Experiment* [12]	Rayleigh number: Theory
0·25	0·5	37 000	28 452
0·25	1·0	15 400	11 962
0·25	2·0	13 600	11 020
0·25	3·675	13 200	10 900

* The lateral walls were slightly conducting.

twenty per cent lower than the measurements. In light of the fact that the lateral walls were slightly conducting, the comparison is very good.

V. SUMMARY

The Galerkin method, which for the present problem is equivalent to a Rayleigh–Ritz procedure was applied to the problem of onset of convection in a confined region. The benefits of using trial functions formed from a linear combination of a complete set of orthogonal coordinate functions has been demonstrated.

Critical Rayleigh numbers were determined for a wide range of aspect ratios and an approximate solution for some ranges of aspect ratio were obtained. Comparison was made with several investigators and in some cases limitations of preceding analysis pointed out. Comparison with experiment was found to be very good.

ACKNOWLEDGEMENT

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APPENDIX

Trial Functions

The velocity components u, v and w are expanded in terms of beam functions to satisfy two boundary conditions at each solid surface. The beam functions are:

$$C_m(x) = \frac{\cosh(\lambda_m x)}{\cosh(\lambda_m/2)} - \frac{\cos(\lambda_m x)}{\cos(\lambda_m/2)} \tag{A.1}$$

with

$$C_m(\tfrac{1}{2}) = C'_m(\tfrac{1}{2}) = 0 \text{ at } |x| = \tfrac{1}{2} \tag{A.2}$$

and

$$S_m(x) = \frac{\sinh(\lambda_m x)}{\sinh(\lambda_m/2)} - \frac{\sin(\lambda_m x)}{\sin(\lambda_m/2)} \tag{A.3}$$

with

$$S_m(\tfrac{1}{2}) = S'_m(\tfrac{1}{2}) = 0 \text{ at } |x| = \tfrac{1}{2} \tag{A.4}$$

where the prime denotes differentiation with respect to the independent variable. These functions and integrals of various combinations of them are found in Harris and Reid [19].

The beam functions are used to generate odd or even number of y rolls. An x roll is a roll about an axis parallel to the y axis. The temperature field is generated with sines and cosines maintaining the same symmetry as that of the vertical velocity component.

The various sets of functions used are as follows:

1. Odd number of x rolls

$$w_{pqr} = \frac{C'_p(x/H_1)}{\lambda p} \cos\left[(2q-1)\pi\frac{y}{H_2}\right] C_r(z) \tag{A.5}$$

$$u_{pqr} = \frac{H_1\lambda_r}{\lambda_p} C_p\left(\frac{x}{H_1}\right)\cos\left[(2q-1)\pi\frac{y}{H_2}\right]\frac{C'_r(z)}{\lambda_r} \tag{A.6}$$

$$T_{pqr} = \sin\left[(2p-1)\frac{x}{H_1}\right]\cos\left[(2q-2)\pi\frac{y}{H_2}\right] \cos\left[(2r-1)\pi z\right] \tag{A.7}$$

2. Even number of x rolls

$$w_{pqr} = \frac{S'_p(x/H_1)}{\mu_p} \cos\left[(2q-1)\pi \frac{y}{H_2}\right] C_r(z) \quad \text{(A.8)}$$

$$u_{pqr} = -\frac{H_1\lambda_r}{\mu_p} S_p\left(\frac{x}{H_1}\right) \cos\left[(2q-1)\pi \frac{y}{H_2}\right] \frac{C'_r(z)}{\lambda_r} \quad \text{(A.9)}$$

$$T_{qpr} = \cos\left(2p\pi \frac{x}{H_1}\right) \cos\left[(2q-2)\pi \frac{y}{H_2}\right] \cos\left[(2r-1)\pi z\right] \quad \text{(A.10)}$$

3. Odd number of y rolls

$$w_{pqr} = \cos\left[(2p-1)\pi \frac{x}{H_1}\right] \frac{C'_q(y/H_2)}{\lambda_q} C_r(z) \quad \text{(A.11)}$$

$$u_{pqr} = -\frac{H_2\lambda_r}{\lambda_q} \cos\left[(2p-1)\pi \frac{x}{H_1}\right] C_q\left(\frac{y}{H_2}\right) \frac{C'_r(z)}{\lambda_r} \quad \text{(A.12)}$$

$$T_{pqr} = \cos\left[(2p-2)\pi \frac{x}{H_1}\right] \sin\left[(2q-1)\pi \frac{y}{H_2}\right] \cos\left[(2r-1)\pi z\right] \quad \text{(A.13)}$$

4. Even number of y rolls

$$w_{pqr} = \cos\left[(2p-1)\pi \frac{x}{H_1}\right] \frac{S'_q(y/H_2)}{\mu_q} C_r(z) \quad \text{(A.14)}$$

$$u_{pqr} = -\frac{H_2\lambda_r}{\mu_p} \cos\left[(2p-1)\pi \frac{x}{H_1}\right] S_q\left(\frac{y}{H_2}\right) \frac{C'_r(z)}{\lambda_r} \quad \text{(A.15)}$$

$$T_{pqr} = \cos\left[(2p-2)\pi \frac{x}{H_1}\right] \cos\left(2q\pi \frac{y}{H_2}\right) \cos\left[(2r-1)\pi z\right]. \quad \text{(A.16)}$$

L'EFFET DE PAROIS VERTICALES ISOLANTES SUR L'INITIATION DU MOUVEMENT DANS UN FLUIDE CHAUFFÉ PAR LE BAS

Résumé—On étudie l'initiation de la convection naturelle dans un fluide confiné dans un espace rectangulaire limité en haut et en bas par des surfaces rigides parfaitement conductrices et latéralement par des parois verticales rigides parfaitement isolantes. Les équations linéarisées de perturbation sont mises sous une forme appropriée sans dimension et sont traitées par un problème à valeur propre. Le nombre de Rayleigh est la valeur propre et est une fonction de deux rapports de forme (largeur/hauteur et profondeur/hauteur). Le problème associé satisfaisant à des conditions aux limites de non glissement sur toutes les surfaces est résolu à l'aide de la méthode de Galerkin. Des résultats sont obtenus pour des rapports de forme compris entre 1/8 et 12. Les résultats sont comparés avec l'expérience et sont trouvés être en bon accord avec elle.

DER EINFLUSS VON ISOLIERENDEN SENKRECHTEN WÄNDEN AUF DAS EINSETZEN DER FREIEN KONVEKTION IN EINEM VON UNTEN BEHEIZTEN FLUID

Zusammenfassung—Der Beginn der freien Konvektion in einem Fluid, das oben und unten durch absolut leitfähige Oberflächen und seitlich durch vollkommen isolierende senkrechte Wände begrenzt ist, wodurch eine rechteckige Form geschaffen ist, wurde untersucht. Die linearisierten Störungsdifferentialgleichungen wurden gewonnen und durch zweckmässiges Dimensionslosmachen zu einem Eigenwert-Problem reduziert. Die Rayleigh-Zahl ist der Eigenwert. Sie ist eine Funktion von zwei Seitenverhältnissen (Breite/Höhe und Tiefe/Höhe). Das Problem, verbunden mit zufriedenstellenden Haftrandbedingungen an allen Oberflächen, wurde durch Anwendung der Galerkin-Methode überwunden. Ergebnisse für ein Seitenverhältnisbereich von $\frac{1}{8}$ bis 12 wurden gewonnen. Die Ergebnisse wurden mit Experimenten verglichen und finden sich mit diesen in guter Übereinstimmung.

ВЛИЯНИЕ ИЗОЛИРОВАННЫХ ВЕРТИКАЛЬНЫХ СТЕНОК НА НАЧАЛО ДВИЖЕНИЯ В ЖИДКОСТИ, НАГРЕВАЕМОЙ СНИЗУ

Аннотация—Исследуется начало естественной конвекции в жидкости, ограниченной снизу и сверху твёрдыми, идеально проводящими поверхностями, а сбоку идеально изолированными вертикальными стенками, вместе образующими прямоугольник. Получены линеаризованные уравнения возмущений, которые сведены к задаче на собственные значения путём соответствующего обезразмеривания. Число Релея представляет собой собственное значение и является функцией двух определяющих отношений (ширина/высота и глубина/высота). Задача, связанная с выполнением граничных условий отсутствия скольжения на всех поверхностях, решается методом Галеркина. Результаты получены для определяющих отношений от 1/8 до 12. Сранение с экспериментальными данными даёт хорошие результаты.